PERIODICITIES OF CERTAIN KEY CIRCULATION INDICES

BY

ROBERT JAY BRAZZELL

PERIODICITIES OF CERTAIN
KEY CIRCULATION INDICES

R. J. Brazzell

[illegible]

[illegible]

[illegible]

PERIODICITIES OF CERTAIN KEY CIRCULATION INDICES

by
Robert Jay Brazzell
Lieutenant junior grade, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
IN AEROLOGY

United States Naval Postgraduate School
Monterey, California
1950

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE
IN AEROLOGY

from the
United States Naval Postgraduate School

PREFACE

The purpose of this research was to investigate statistically the relationship between certain key circulation indices and to determine the nature of their periodicities.

This work was conducted at the United States Naval Postgraduate School, Monterey, California, during the period December 1949 to May 1950. It was done to meet partial requirements for the degree of Master of Science in Aerology. The idea for this investigation was suggested by Professor F. L. Martin of the staff of the Department of Aerological Engineering of the United States Naval Postgraduate School. His assistance and advice in the preparation of this thesis is gratefully acknowledged.

TABLE OF CONTENTS

	Page
CERTIFICATE OF APPROVAL	i
PREFACE	ii
TABLE OF CONTENTS	iii
LIST OF ILLUSTRATIONS	iv
TABLE OF ABBREVIATIONS	v
CHAPTER	
I. INTRODUCTION	1
II. METHOD OF ANALYSIS	3
III. RESULTS AND INTERPRETATIONS	8
BIBLIOGRAPHY	29
APPENDICES	
I. Monthly Mean Hemispheric Surface Temperatures at 45° Latitude in Degrees Absolute	30
II. The Periodogram	32

LIST OF ILLUSTRATIONS

		Page
Figure 1.	Trend Curve of Zonal Index	15
Figure 2.	Trend Curve of Meridional Index	16
Figure 3.	Trend Curve of 500 mb Zonal Index	17
Figure 4.	Trend Curve of Thermal Wind Index	18
Figure 5.	Periodogram of Zonal Index	24
Figure 6.	Periodogram of Meridional Index	25
Figure 7.	Periodogram of Thermal Wind Index	26
Figure 8.	Periodogram of Regular Sine Wave	31

PLATE

(inside back cover)

I. Timegraph of Zonal Index

II. Timegraph of Meridional Index

III. Timegraph of 500 mb Zonal Index

IV. Timegraph of Thermal Wind Index

TABLE OF ABBREVIATIONS

ZI	Sea level zonal index
MI	Sea level meridional index
TWI	Thermal wind index
ZI_{500}	500 mb zonal index
HTI	Heat transport index

I. INTRODUCTION

It is a well known meteorological fact that the accurate long range forecast is impossible until an adequate understanding of the erratic fluctuations of the global circulation indices is obtained.

The fluctuations, although showing certain "trends" that can be forecast with a fair degree of accuracy for relatively short periods, are of such irregularity that, as yet, no periodicity has been established.

Recent investigations conducted by H. C. Willett [10] have indicated that correlations between selected indices used to express the state of the general circulation are not highly significant. These correlations, computed for each of seven selected half years, averaged -0.30 for Zonal with meridional flow and -0.32 for Zonal flow with the poleward gradient of mean virtual temperature. These correlations show little if any significant relationship. He attributes these poor correlations to the incompleteness of available synoptic data, the crudeness of the correlation technique for the expression of statistical relationships, and the interference which is offered by the continent of Asia.

The purpose of the investigation was twofold:

(1) To substantiate or disprove Willett's results concerning the statistical relationships between these indices by using their daily values in preference to their five day means.

(2) To determine possible reasons for the poor correlations reported by Willett if this investigation proved the validity of his results.

Accomplishment of the second objective has led to an investigation of possible periodicities in the several branches of the general circulation. For this purpose, the technique of the periodogram, described in Appendix II, has been utilized.

The chief results determined in this thesis indicate that:

(a) There are no strongly significant correlations between the general circulation indices herein considered. In general the results are opposite in sign to those obtained by Willett with approximately the same order of significance.

(b) One of the most significant correlations indicates a thermal basis of the general circulation. It is found that both the zonal and meridional indices show a positive correlation with the poleward gradient of mean virtual temperature.

(c) The observed weakly significant linear correlations do not necessarily indicate a lack of relationship between indices.

(d) Periodicities suitable for use in a harmonic analysis were determined for each index. These results indicate a basic periodicity of 12 to 14 days.

II. METHODS OF ANALYSIS

This chapter describes in detail the source of data, definitions, and methods of computation of all indices and, in the case of the HTI, the reason for computation as well as the method.

All indices were computed on a daily basis for the entire northern hemisphere, chiefly because of a comment by H. C. Willett [9] that, "daily index correlations show a tendency to be more significant for the entire hemisphere than for any individual quadrant, which suggests that even the daily fluctuations of the general circulation pattern are interrelated on a worldwide scale insofar as there are any consistent patterns of behavior and that the operation of any such interrelation as exists is not primarily local in character."

Also three day running means were used in preference to the actual daily values in order to (1) damp out observational errors; (2) eliminate very small insignificant variations and sharpen the maximum and minimums.

The source of observational information was the Northern Hemisphere Historical Weather Series [4] of analyzed sea level and 500 mb weather maps. The months of October 1945 to March 1946 (182 daily values) were used for this study.

Definitions and methods of computation of the indices:

1. Zonal index (ZI) This index is defined as the average sea level pressure difference in millibars between latitudes 35° and 55°. The difference in pressure between these latitudes was taken at each 10° of

[illegible]

pressure difference [illegible]

difference in pressure [illegible]

longitude and the arithmetic mean computed. This mean was taken as the value of that day's index and is probably accurate to 0.1 millibars.

2. Meridional index (MI) The conventional method of computing this index was not used because it fails to account for meridional flow between two adjacent 10° longitudes with identical pressures although there is a variation in pressure between them; i.e., a sharp trough centered between two adjacent longitude lines on a weather map would give no contribution to the north-south flow when computed by the conventional method so the following procedure was used. The pressure was recorded at each point of maximum or minimum pressure around the 45th parallel. The difference in pressure between adjacent maximum and minimum was computed and the sum of their absolute values was obtained. This sum was taken as the true reflection of the total meridional geostrophic flow across the 45° latitude circle. This sum was divided by thirty-six, so the results could be compared to the meridional index computed by the regular method. The units of this index are also millibars and its computation is considered to be accurate to 0.1 millibar.

3. 500 mb Zonal index (ZI_{500}) This index is computed in exactly the same way as ZI, except that height differences in hundreds of feet are obtained instead of pressure differences. These differences reflect the intensity of the geostrophic zonal circulation, between the specified latitudes, at this level. These values are considered accurate to 5 feet.

4. Thermal wind index (TWI) Since the mean zonal geostrophic winds at both sea level and 500 mb are uniquely determined by their respective indices, we could obtain, by simply subtracting these geostrophic values the mean zonal thermal wind. This mean zonal thermal wind is in turn proportional to the meridional gradient of mean virtual temperature.

In order to simplify the computation of the thermal wind index, it was decided to use an index obtainable directly from the sea level and 500 mb zonal indices, respectively, without the necessity of conversion into geostrophic winds. Thus the values of sea level zonal index were converted into values of a zonal index measured on the 1000 mb pressure surface (ZI_{1000}). Thermal wind index is then defined as

$$(1) \quad TWI = ZI_{500} - ZI_{1000}$$

and is directly proportional to the mean virtual temperature gradient between 35° and 55° latitude for the layer 500-1000 mb.

It was next necessary to consider the method of converting sea level zonal index in millibars into ZI_{1000} in feet. From the integrated form of the hydrostatic equation it follows that:

$$(2) \quad ZI_{1000} = \frac{R}{g}\left\{\left(\frac{\bar{T}}{P}\right)_{\phi}\left(\bar{P}_{35} - \bar{P}_{55}\right) + \left(\bar{P}_{45} - 1000\right)\left[\left(\frac{\bar{T}}{P}\right)_{35} - \left(\frac{\bar{T}}{P}\right)_{55}\right]\right\}$$

where

R is the gas constant per unit mass of air

g is the acceleration of gravity

$(\bar{T}/P)_{\phi}$ is the average value at latitude ϕ of the ratio of absolute temperature to pressure in millibars, both measured at sea level.

$\overline{P}_{\phi}$ is the average values of sea level pressure at latitude ϕ ($\phi = 35^{\circ}$ 45° or 55°)

As an example, on 15 October 1945 the second term of the right side of (2) amounts to 10 feet as compared to 180 feet the average value of the first term of the right side of (2). Thus within an error of approximately 10 feet, equation (2) may be simplified to

$$(3) \quad ZI_{1000} = .096(\overline{T})_{45}\ (ZI)$$

where

$(\overline{T})_{45}$ is the mean temperature, at sea level, at latitude 45° for the day under consideration.

Actually a year-round average temperature $\overline{T}$ could have been used in (3). This would be equivalent to using the familiar substitution 7.5 millibars corresponding to 205 feet. However, in order to eliminate as much further error as practicable, it was decided to use the monthly mean temperatures $\overline{T}_{45}$ in place of the daily mean temperature at latitude 45°, a procedure accurate within 1% of the true value of (3). It therefore follows that ZI_{1000} given by (3) computed using $\overline{T}_{45}$ is accurate to within 15 feet. Since ZI_{500} was considered accurate to 5 feet, the resultant TWI is accurate to 20 feet; that is, to $.25^{\circ}$ centigrade.

5. Heat transport index (HTI) This index was computed in order to investigate a theory due to Starr but stated by H. C. Willett [11]

> from certain theoretical reasoning concerning the thermodynamic mechanism of the latitudional transport of heat and kinetic energy Professor Victor Starr comes to the conclusion that there should be operative in this transport mechanism an exclusion principle such that an extensive poleward transport of these properties should be suppressed at any middle latitude poleward from which there exists

Actual [illegible]

This [illegible]

correct [illegible]

error [illegible]

T_{45} [illegible]

accur[illegible] of [illegible]

Z_{1000} [illegible] (7) [illegible]

Since [illegible]

accurate to [illegible] feet; that [illegible] 45° [illegible]

5. Heat transport index (H.T.). This index [illegible]

vestigate a theory due to [illegible] but started [illegible]

From certain theoretical reasoning concerning the [illegible] mechanism of the latitudinal transport of heat and [illegible] Professor Victor Starr comes to the conclusion that [illegible] operative in this transport mechanism an exclusion principle such that an extensive poleward transport of these properties should be suppressed at any middle latitude poleward from which there exists

a marked deficit of sea level pressure; i.e., a strong circumpolar cyclonic vortex or strong zonal westerlies should necessarily repress such transport.

Professor Starr correlated 26 daily values of sea level pressure deficit and heat transport with a resulting correlation of -0.60 as a test for his hypothesis.

In order to tentively investigate this hypothesis with data at hand, it was assumed that the Zonal Index was a valid index of the deficit of pressure poleward of 45° north latitude and that the product of the surface MI and the TWI is an approximate index of the heat being transported across the 45° latitude circle. This product is defined as the heat transport index. (HTI)

6. Other tools

a. Trend curves - These curves were obtained by calculating the 31-day mean of the variate for each week of the period under consideration, and plotting them against time. Thus the slope and direction of the curve will yield the sign and the magnitude of the trend at any particular time while the deviation of the curve from the average value of the variate, for the complete period, will indentify periods of relatively high and relatively low values.

b. Periodogram - This is a method of determining the approximate periods of regular waves from a curve which is essentially a sum of such regular waves. For a complete explanation of this method refer to Appendix II.

III. RESULTS AND INTERPRETATIONS

1. Time Graphs of Daily Index Values.

Plates I, II, III, IV show time graphs of the variation of the indices for the 182 days under investigation. Study of these graphs show: a close similarity between the ZI curve and that of the ZI_{500}; the ability of the zonal index to change its character from very high to very low in as little as four days; a possible negative relationship between the daily values of ZI and those of TWI; and the apparently complete independence of the daily variation of the meridional index in relation to any of the other indices. Linear correlations have been computed between daily values of the various indices and are listed in tables which follow.

2. Comparison with Willett's Results.

Comparison of results obtained in this investigation with those obtained by Willett [10] using five day mean values ranging over seven different winters, indicate that the five day means are not an accurate reflection of the daily variation of the indices. For instance, Willett's correlation of ZI with MI gave a moderately significant negative coefficient whose average value was -0.30 while our correlation of the daily values resulted in a coefficient of 0.13. Similarly, Willett found a significant negative coefficient of -0.30 when he correlated the poleward gradient of mean virtual temperature with ZI, while the daily values showed an insignificant positive relationship of 0.05. However, his result of 0.90 for a seven season average between ZI and ZI_{700} is in good agreement with the 0.70 coefficient obtained by the correlation of the daily values of ZI

TABLE I

Other Variate	Number of Values	Significant Coefficient for 1% level of belief	Calculated Coefficient	Degrees of Freedom Used	Remarks
MI	182	0.27 0.33	0.13	91 60	Daily values
MI	71	0.42 0.49	0.03	35 24	High values ZI > 8 mb
MI	48	0.49 0.59	-0.21	24 16	Low values ZI < 4 mb
MI	36	0.56 0.66	0.28	18 12	Very high values ZI > 10 mb
MI	63	0.43 0.52	0.02	32 21	Normal values 4 mb < ZI < 8 mb
MI	24	0.50	0.28	22	31-day means
ZI_{500}	182	0.27 0.33	0.70	91 60	Daily values
ZI_{500}	24	0.50	0.79	22	31-day means
TWI	24	0.50	0.36	22	31-day means
TWI	182	0.27 0.33	0.05	91 60	Daily values
HTI	182	0.27 0.33	0.08	91 60	Daily values
HTI	44	0.50 0.61	-0.78	22 15	Selected data Oct 17-Nov 30
HTI	58	0.45 0.54	0.40	29 19	Selected data Jan 9-Mar 8

Zonal Index Correlations

TABLE II

Other Variate	Number of Values	Significant Coefficient for 1% level of belief	Calculated Coefficient	Degrees of Freedom Used	Remarks
ZI	182	0.27 0.33	0.13	91 60	Daily values
ZI	71	0.42 0.49	0.03	35 24	$ZI > 8$ mb
ZI	48	0.49 0.59	-0.21	24 16	$ZI < 4$ mb
ZI	36	0.56 0.66	0.28	18 12	$ZI > 10$ mb
ZI	63	0.43 0.52	0.02	21	$4 < ZI < 8$ mb
ZI	24	0.50	0.28	22	31-day means
ZI_{500}	182	0.27 0.33	0.17	91 60	Daily values
ZI_{500}	24	0.50	0.73	22	31-day means
TWI	182	0.26 0.33	0.42	91 60	Daily values
TWI	24	0.50	0.11	22	31-day means

Meridional Index Correlations

TABLE III

THERMAL WIND INDEX

Other Variate	Number of Values	Coefficient for .01 level belief	Calculated Coefficient	Degrees of Freedom	Remarks
ZI	182	0.27 / 0.33	0.05	91 / 60	Daily values
ZI	24	0.50	0.36	22	31-day mean
MI	182	0.27 / 0.33	0.42	91 / 60	Daily values
MI	24	0.50	0.71	22	31-day mean

HEAT TRANSPORT INDEX

Other Variate	Number of Values	Coefficient for .01 level belief	Calculated Coefficient	Degrees of Freedom	Remarks
ZI	182	0.27 / 0.33	0.08	91 / 60	Daily values
ZI	44	0.50 / 0.61	-0.78	22 / 15	Selected data Oct 17-Nov 30
ZI	58	0.45 / 0.54	0.40	29 / 19	Selected data Jan 9-Mar 8

Multiple Correlations

ZI = 1	$r_{1:23} = 0.135$
MI = 2	$r_{2:31} = 0.437$
TWI = 3	$r_{3:12} = 0.344$?

Thermal Wind, Heat Transport and Multiple Correlations

and ZI_{500}. Willett's results showed no significant lag correlations over periods of three to four weeks. This result is verified in this investigation with the exception of the correlation of MI with TWI which shows a contempary correlation of 0.424.

As a coefficient of 0.32 is considered significant on the 1% level of belief* for variates with only sixty degrees of freedom, it would seem that those just mentioned, derived from the correlation of 150-180 reports, must be considered significant. These results strongly indicate a thermal basis of the general circulation. Thus an increase in TWI to above normal values represents an increased concentration of solenoids which therefore gives rise to increased MI. This, in turn, is in general associated with an increase of ZI.

3. Trend Curves and Correlations.

The trend curves were constructed for the four indices in order to investigate the waves of longer period. Thirty-one day means, taken seven days apart, were used in obtaining these curves. These relatively long-term means effectively block out the shorter waves leaving the longer period "trend" curve free for investigation. The resultant curves (Figures 1-4) show that all the indices increased during the winter season to a maximum in the latter part of January. The MI trend curve, although well above the yearly average during the winter months does show a minimum at the time the ZI exhibits its absolute maximum. This leads one to conclude that the increased circulation as evidenced by increasing ZI would also cause increasing north-south flow, and that it is only with very high values of ZI, indicative of an extremely intense zonal circulation, that

*cf Fisher [2], p. 202

[illegible]

of [illegible]

that [illegible]

must [illegible]

north [illegible]

values [illegible]

gives [illegible]

in increase [illegible].

5. [illegible]

[illegible]

investigate the [illegible]

days [illegible], were [illegible] for these curves. [illegible]

means [illegible] the [illegible]

"trend" [illegible] free for [illegible]. The resulting curves (Figure 1-[illegible])

show that [illegible] the [illegible] during the [illegible]

in the latter part of [illegible], [illegible] trend [illegible], although [illegible]

the yearly average during the winter [illegible]

time the SI exhibits its [illegible]. This leads [illegible]

that the increased circulation as evidenced by increasing SI [illegible]

cause increasing north-south flow, and that it is only high [illegible]

values of SI, indicative of an extremely intense zonal circulation, that

*cf Fisher [2], p. 202

the pressure systems tend to be oriented in a west-east direction causing a decrease in meridional flow. The correlation 0.28 between the 31-day means of ZI and MI is probably weakened as a result of this trend change in mid-winter and falls well below the 1% level of significance 0.50 established for this investigation. Further examination of the trend curves showed a marked similarity between the curves for ZI and ZI_{500}. This could have been anticipated by the significant correlation 0.70 between daily values of the two variates and is borne out by the 0.79 coefficient obtained by correlating the 31-day means. These facts seem to indicate that there is a close relationship between the circulation at the surface and that in the upper levels. This of course was to be expected from Willett's results. Visual comparison of the trend curves of MI and TWI show little apparent similarity except for the seasonal rise toward a mid-winter maximum in each, yet correlation of their 31-day means give the significant coefficient of 0.71, again a marked improvement over the coefficient resulting from daily values. Similarly, the correlation for the 31-day means of ZI with TWI, 0.36, shows a considerable increase over that of the daily values 0.05 for the same period. These comparisons point to the trend curves as being more significant indices of the variation of the branches of the general circulation than the daily values. The latter values apparently undergo rather erratic fluctuations about their 31-day mean.

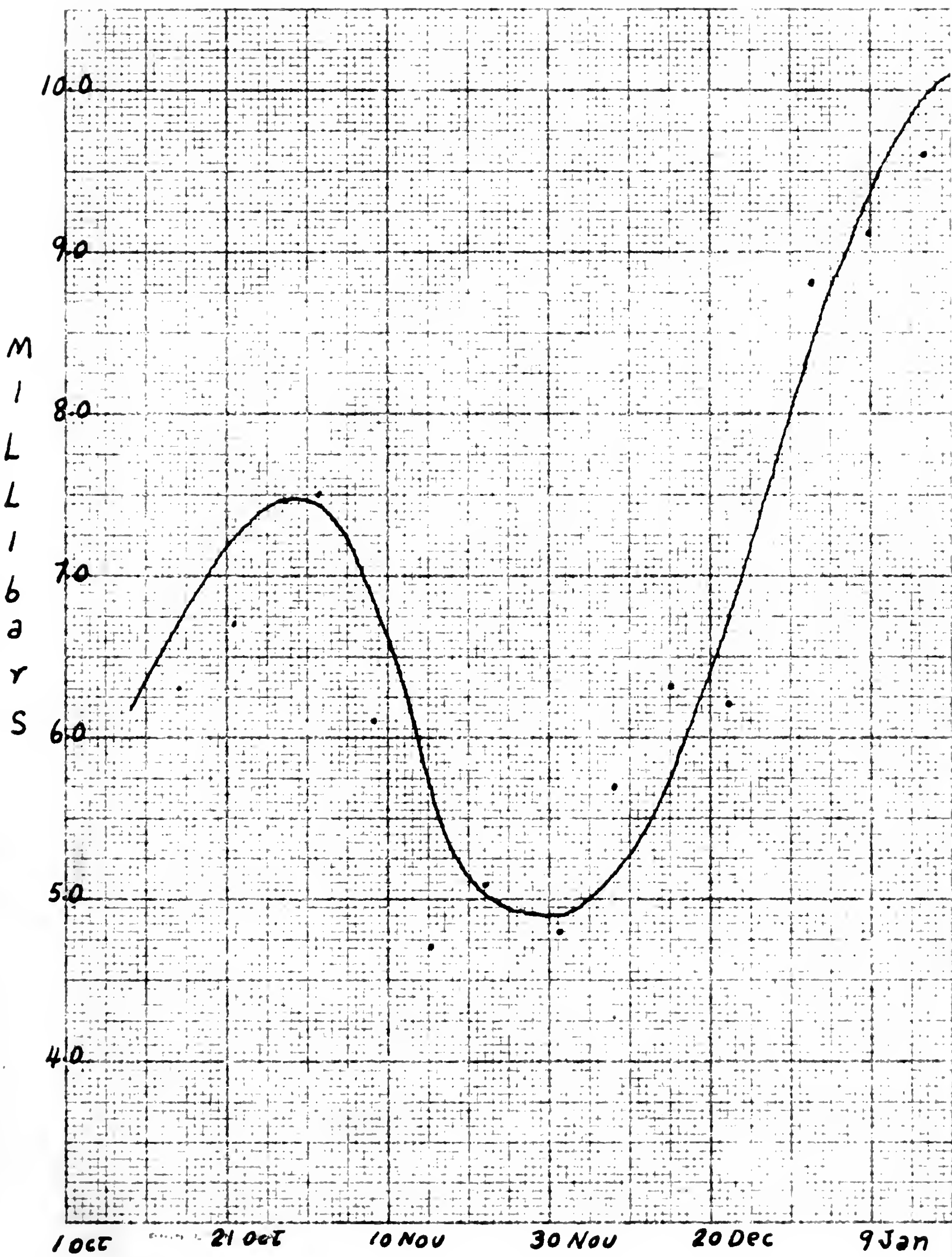
4. Effect of the Intensity of the Zonal Circulation on the Meridional Flow.

As stated previously the overall seasonal (trend curve) effect (Figures 1 and 2), is that both ZI and MI increase together to relatively high values in mid-winter. However, these figures also show that during the period when the average ZI is at its absolute maximum, the MI exhibits a minimum. Correlations of the daily values of the two variates on days exhibiting selected values of ZI (see Table I or II) show that for ZI $>$ 10; i.e., a very high ZI, an increase in ZI still results in an increase in meridional flow, in spite of the opposite tendencies of ZI and MI noted above. These two results are not however incompatible.

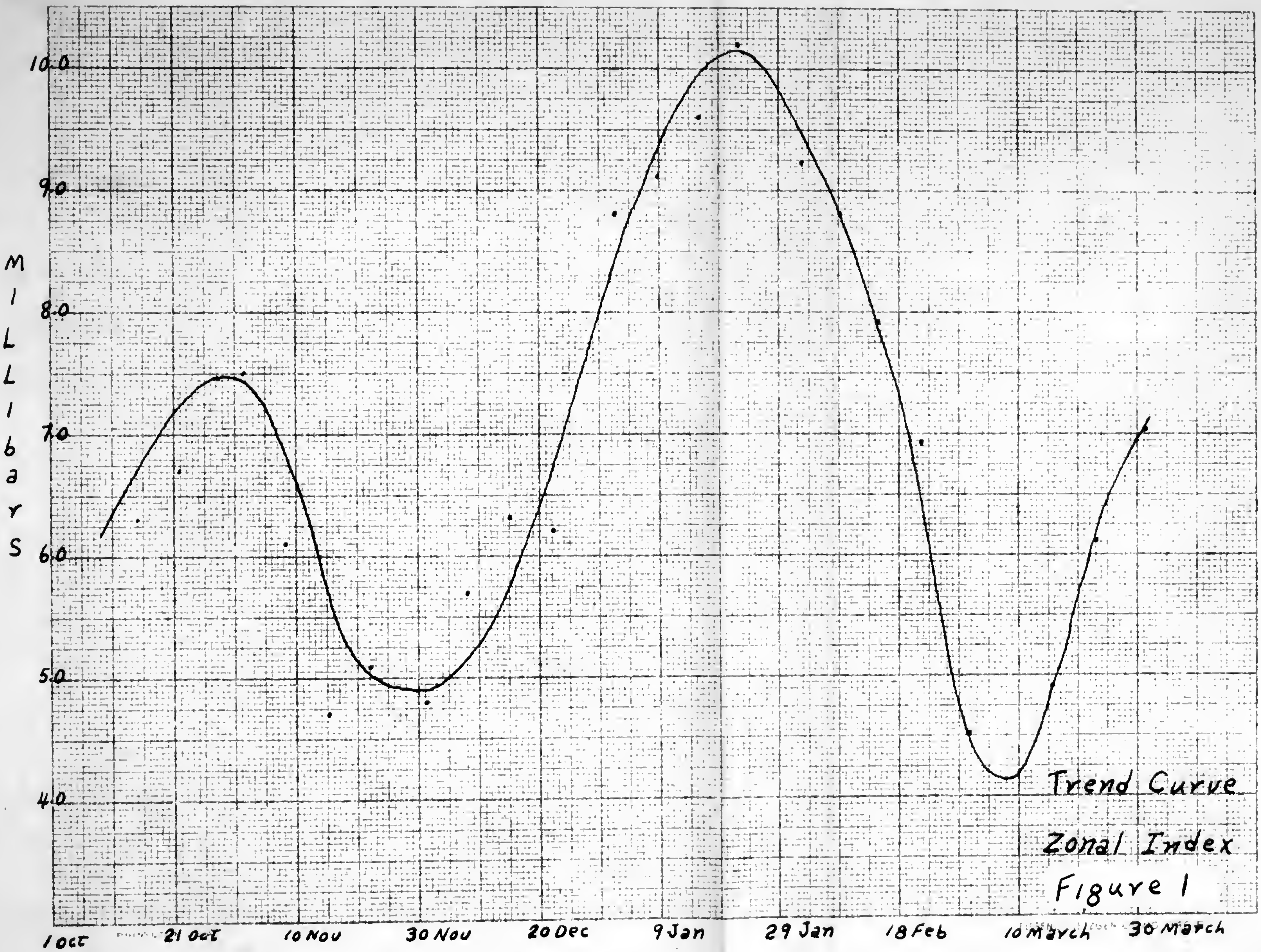
The negative correlation for values of ZI $<$ 4 mb can be explained by recalling that during periods of low index the polar front is in relatively low latitudes causing considerable easterly flow to be introduced in the computation of ZI. Thus a decrease in cyclonic activity along the polar front would cause an increase in the zonal index and a decrease in meridional flow.

5. Conclusions of the Linear Regression Analysis.

In comparing the results of this paper with those of Willett [9], [10] certain differences in technique should be taken into consideration. Since his data was spread over seven winters, it was necessary for him to draw an average "seasonal curve" for his individual values from the indicated seasonal value for the particular period. Since the data of the present paper was all from one winter season it was decided to use the deviations from the sample mean as the basis of our correlations. This decision was based on several reasons:



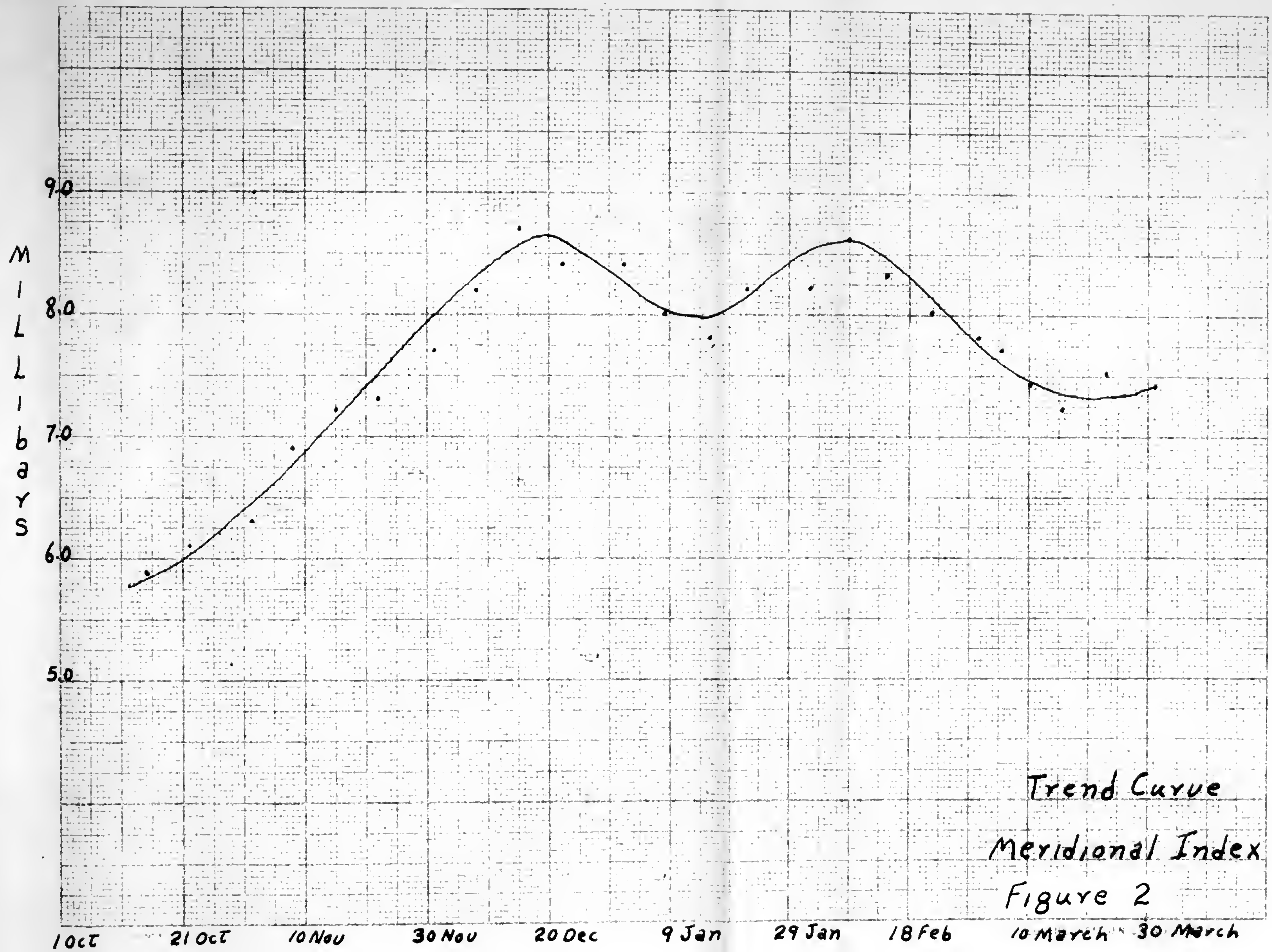
10.0
9.0
8.0
7.0
6.0
5.0
4.0
MILLIbarS
1 oct
21 Oct
10 Nov
30 Nov
20 Dec
9 Jan



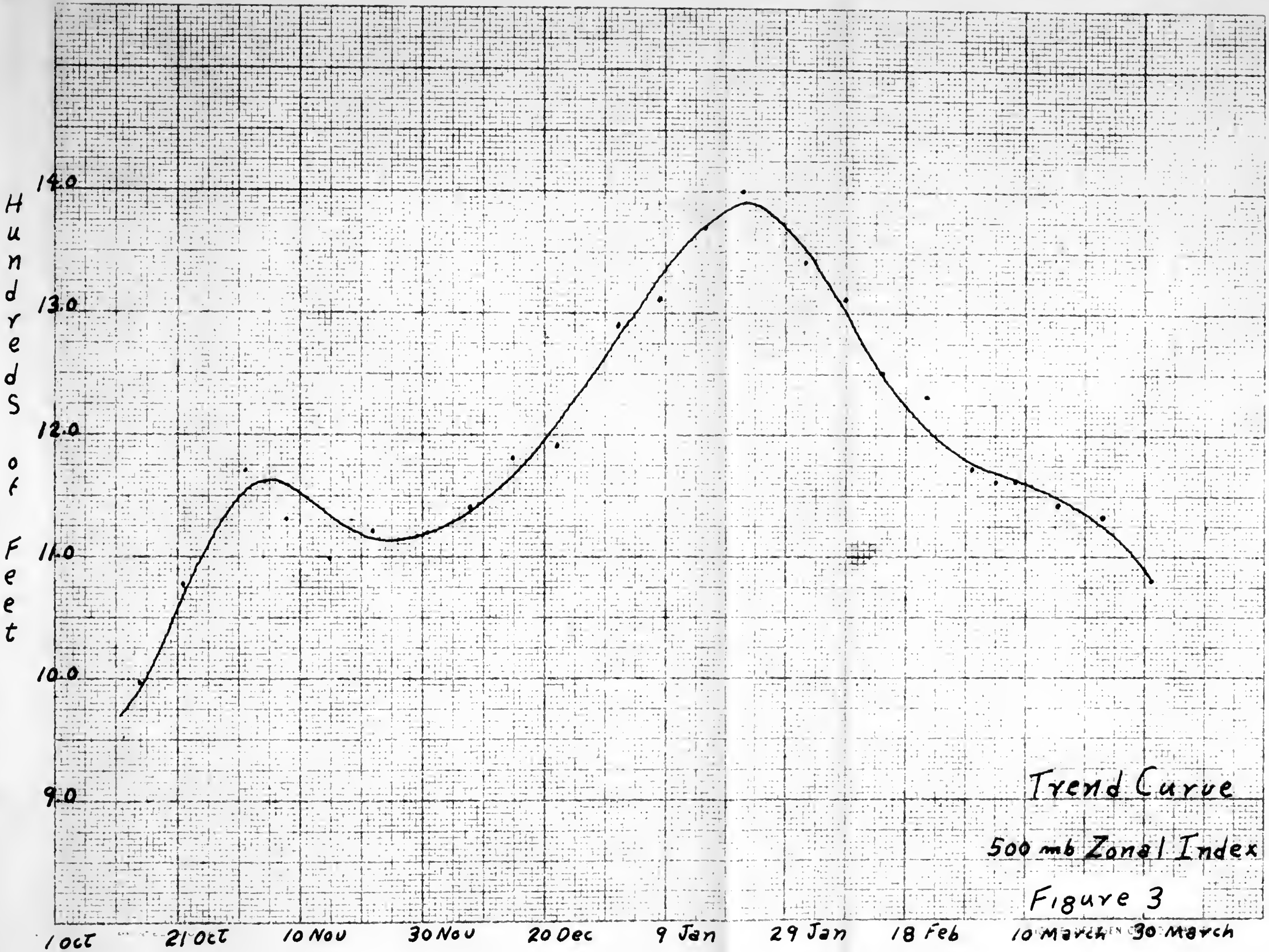
Trend Curve

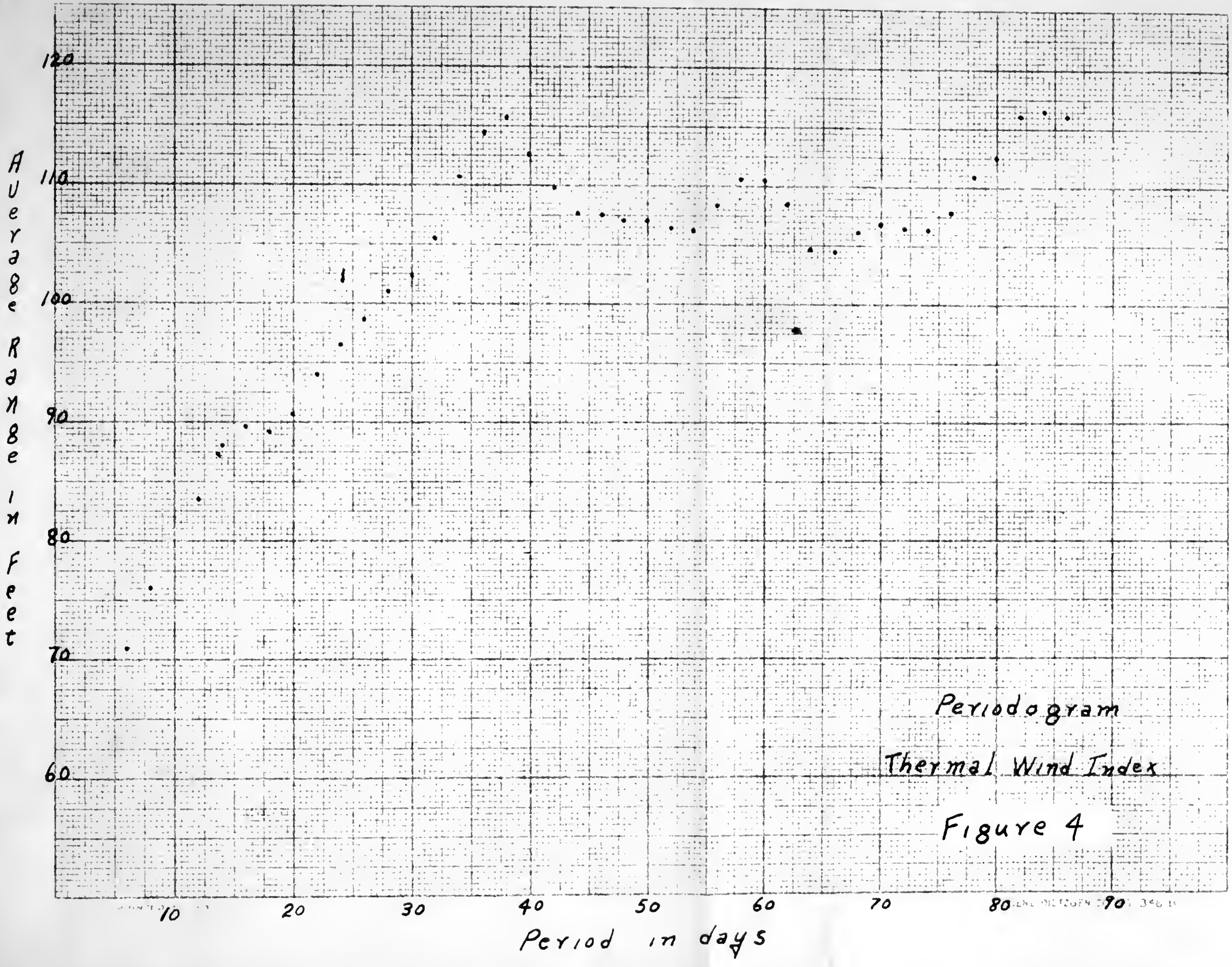
Zonal Index

Figure 1



9.0
8.0
7.0
6.0
5.0
MILLIBARS
1 Oct
21 Oct
10 Nov
30 Nov
20 Dec
9 Jan
29 Jan
18 Feb
10 March
30 March
Trend Curve
Meridional Index
Figure 2


Hundreds of Feet
14.0
13.0
12.0
11.0
10.0
9.0
1 oct
21 Oct
10 Nov
30 Nov
20 Dec
9 Jan
29 Jan
18 Feb
10 March
30 March
Trend Curve
500 mb Zonal Index
Figure 3



Average Range in Feet
120
110
100
90
80
70
60
10
20
30
40
50
60
70
80
90
Period in days
Periodogram
Thermal Wind Index
Figure 4

(a) The trend or "seasonal curves" discussed in Section 3 were based on 31-day means taken seven days apart and were taken initially for the purpose of using them in a manner similar to Willett [9]. These curves showed significant maxima and minima and it was not clear how these should be smoothed over. In fact, there was no unique way to draw the seasonal curve through the indicated points.

(b) The existence of these maxima and minima are manifestations of certain longer period trends and cannot be considered even approximately constant in time from year to year. This was clearly shown by Riehl [7].

(c) A strict comparison of our results with Willett's was carried out. Preliminary correlations based on the differences between daily values of an index and the smoothed values taken from the trend curve were obtained but indicated insignificance in every case.

(d) The results of the trend curve study justified this decision insofar as daily data was concerned. It has already been observed that correlation coefficients computed using the 31-day means of each variate showed in every case the same sign as the one computed for daily values, with a much more significant value.

The chief results of the linear correlations study are summarized below:

(1) In contrast to Willett's results this investigation showed that the daily values of ZI and MI can be expected to increase and decrease together in all cases except periods of low zonal index.

(2) That MI and TWI show a positive correlation for both daily and mean values that is well above the significance level.

These results indicate a thermal basis of the general circulation in contrast to the mechanical basis suggested by Rossby [8]. This raises the question of the relationship between the TWI and MI at upper levels, which might possibly give even more significant results than it does at the surface.

(3) The indices TWI and ZI also show a weakly significant positive correlation when 31-day means are considered. This result is entirely consistent with results 1 and 2, concerning the thermal basis of the general circulation.

6. Correlations of ZI with HTI

The correlations of the zonal index with the previously defined heat transport index, 0.08, was disappointing in that it was believed that the assumptions made were nearly enough correct to give at least the sign of Starr's correlation. However, the results obtained indicate that either the assumptions made were grossly in error or that Starr's premise was incorrect. It is rather interesting to note that Starr's value of -0.60 for the correlation between pressure deficit in high latitudes and meridional heat transport was obtained from twenty-six consecutive daily values of the two variates. In the data under consideration in this thesis a

[illegible]

(5) [illegible]

daily [illegible]

[illegible]

(6) The [illegible] shows [illegible]

positive [illegible] 1-day [illegible] are considered. [illegible] is entirely consistent with results 1 and 2, [illegible] basis of the [illegible] circulation.

6. Correlations of [illegible]

The correlations [illegible] zonal index with [illegible] heat transport index, 0.06, was disappointing in that it was believed that the assumptions made were nearly enough correct to give at least the [illegible] of Starr's correlation. However, [illegible] results obtained indicate that either the assumptions made were grossly in error or that Starr's premise was incorrect. It is rather interesting to note that Starr's value of -0.70 for the correlation between pressure deficit in high latitudes and meridional heat transport was obtained from twenty-six consecutive daily values of the two variates. In the data under consideration in this thesis a

correlation of -0.784 was obtained using values of the variates on forty-four consecutive days in one selected period, while a correlation of 0.40 was obtained using values from another consecutive fifty-eight day period within the same six-month period. It is therefore possible that Starr's results were obtained from a period comparable to our forty-four day period mentioned above. These two differing results seem to indicate that both ZI and HTI are composed of similar harmonic components but slightly different periods. Thus the phase relationship between them would be constant enough to give a good negative correlation during the out of phase cycle. As more and more data were correlated, however, the shifting phase due to differing periodicities, would cause the correlation to weaken, and at some later selected period positive correlations could be obtained. The time required for this "shift of phase" in this particular instance seems to be about ninety days.

An examination of Plates I, II, and IV indicates that the characteristic of varying periodicities is by no means restricted to ZI and HTI, but also applies to MI and TWI. This led to an investigation of the periodicities indicated in the harmonics that comprise the daily index curves of ZI, MI and TWI.

In this connection even a complete harmonic analysis would be of little significance if the basic period and overtones of an index varied radically in a short time beyond the period of study. Nevertheless it seemed possible that such an investigation might reveal some basic periodicity of the atmospheric waves.*

*cf Rossby's "turbulence waves" [8], with periods 4-6 weeks.

7. Periodicities

The periodogram technique, discussed in Appendix II, is a method of obtaining the favored periodicity from data in which there are several superimposed harmonic components, one of basic period and its overtones. In this analysis, periodograms were constructed for the zonal index, meridional index and the thermal wind index (Figures 5, 6, and 7). From an examination of each of these three periodograms we see that there are several more or less regularly-spaced maxima, which strongly suggests a repeating cycle. Let us consider in turn each of the periodograms obtained.

(a) Zonal Index.- This periodogram shows maximum ranges at 14, 36, 54 and 80 days. Using the initially indicated period of 14 days together with its overtones it was possible to explain the thirty-six day maximum as the interference effect of the superposition of the 28 and 42 day harmonics. However, the combination of amplitudes necessary for this 36-day maximum was not compatible with a maximum at 56 days.

Probably the best qualitative solution can be obtained by noting that the average time in days between observed maxima is 22-24 days. This, as previously mentioned, suggests repeated indications of the same periodicity. By closer examination we see that the best possible fit of the observed maxima will be obtained by introducing a basic period of 12 days which would exhibit maximum ranges at 12, 36, 60 and 84 days. Now introduce the odd overtones of this basic period. These would exhibit maximum at 36, 60 and 84 days and would explain the increased range between the first and second observed maximum of the ZI periodogram. When it is remembered that the periodogram of an ideal wave form is rather flat in the vicinity of its maximum (Figure 8), we can see how the effect of the contribution of the

[illegible]

[illegible]

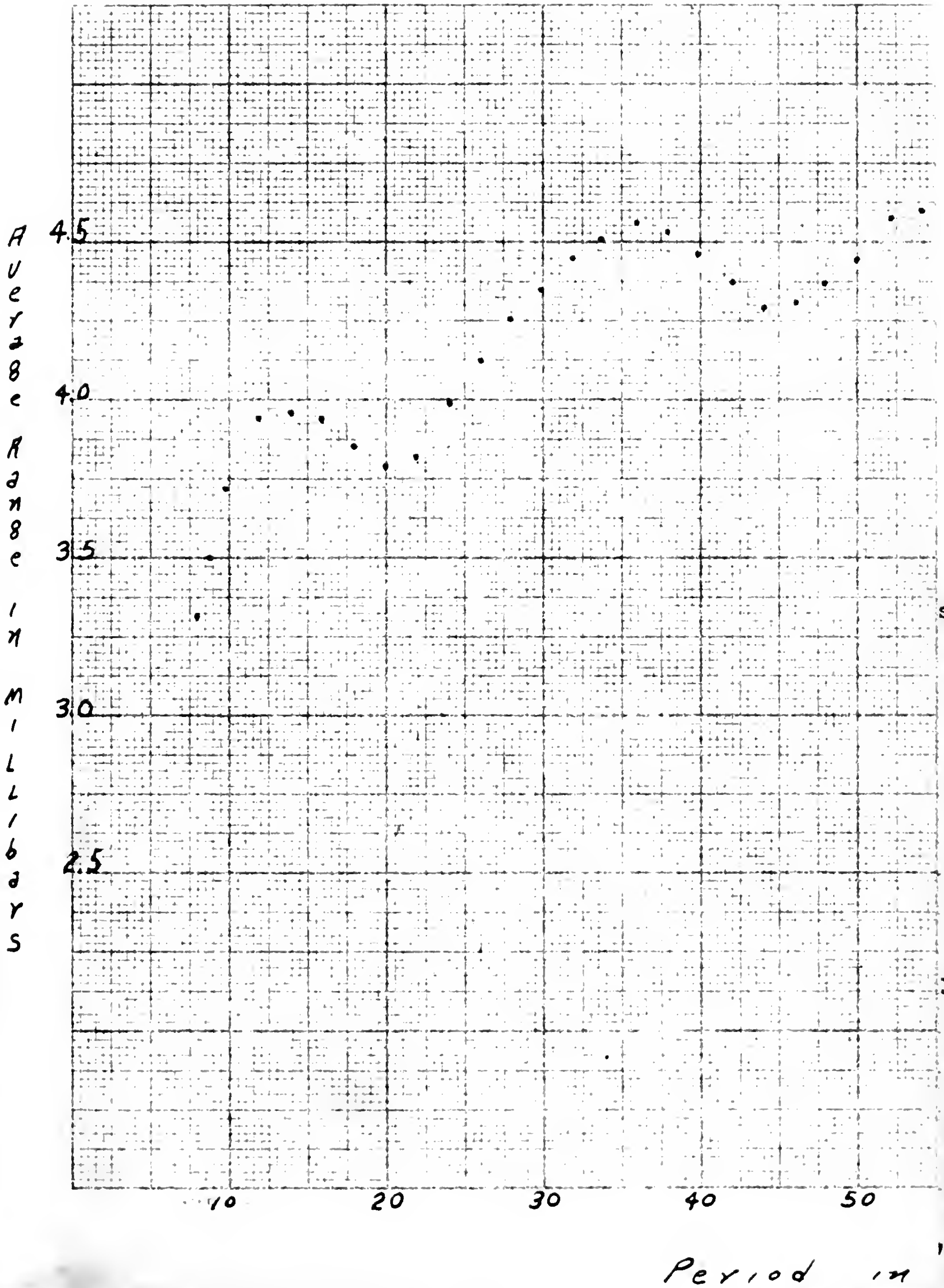
Probably the most satisfactory explanation [illegible] the average time in days between observed maxima is 28–34 days. This, as previously mentioned, [illegible] in the [illegible] By closer examination we see that the best possible fit of the observed maxima will be obtained by introducing a basic period of 12 days which would exhibit maximum ranges at 12, 36, 60 and 84 days. Now introduce [illegible] overtones of this basic period. These would exhibit maxima at 36, 60 and 84 days and would explain the increased range between the first and second observed maximum of the 21 periodogram. When it is remembered that the periodogram of an ideal wave form is rather flat in the vicinity of its maximum (Figure 8), we can see how the effect of the combination of the

ranges of longer period waves could cause the first maximum to be shifted by several days in the direction of the longer periods. This is the reason that the periodicity that best fits all maxima was selected. In addition, the one-day interval between values of the variate makes the closest definition of a basic period, by this type of analysis, accurate only to the nearest two-day interval.

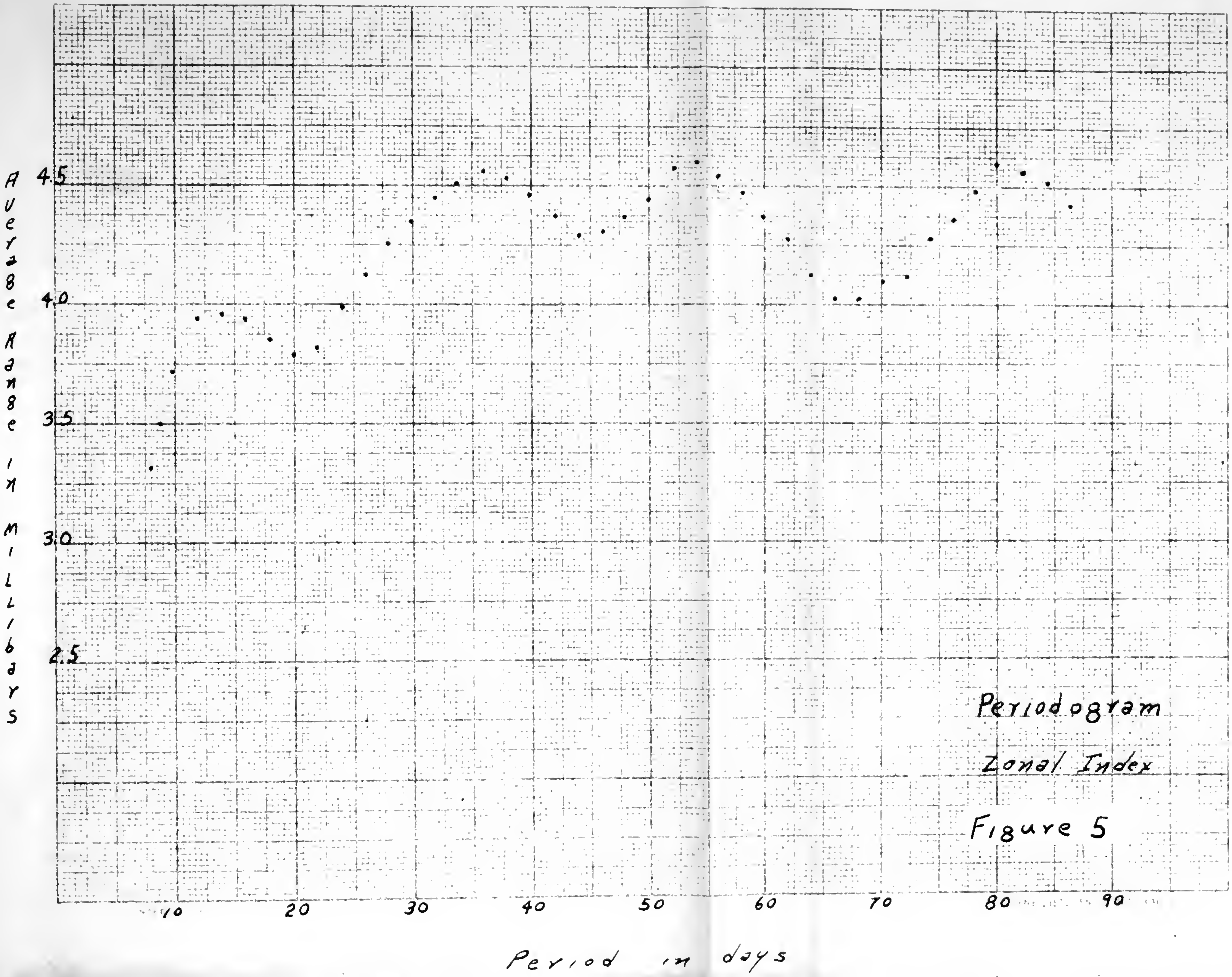
Another possibility is that additional range over and above the recurring basic 12 day wave contribution could be more or less uniform throughout the periods 30-80 days, and therefore not indicative of overtones. However, the harmonic analysis viewpoint prescribes the overtones; i.e., both even and odd multiples of the basic period. At any rate a basic period of about 12 days appears to be fairly well established for the ZI and the overtone contribution would have to be determined by a harmonic analysis.

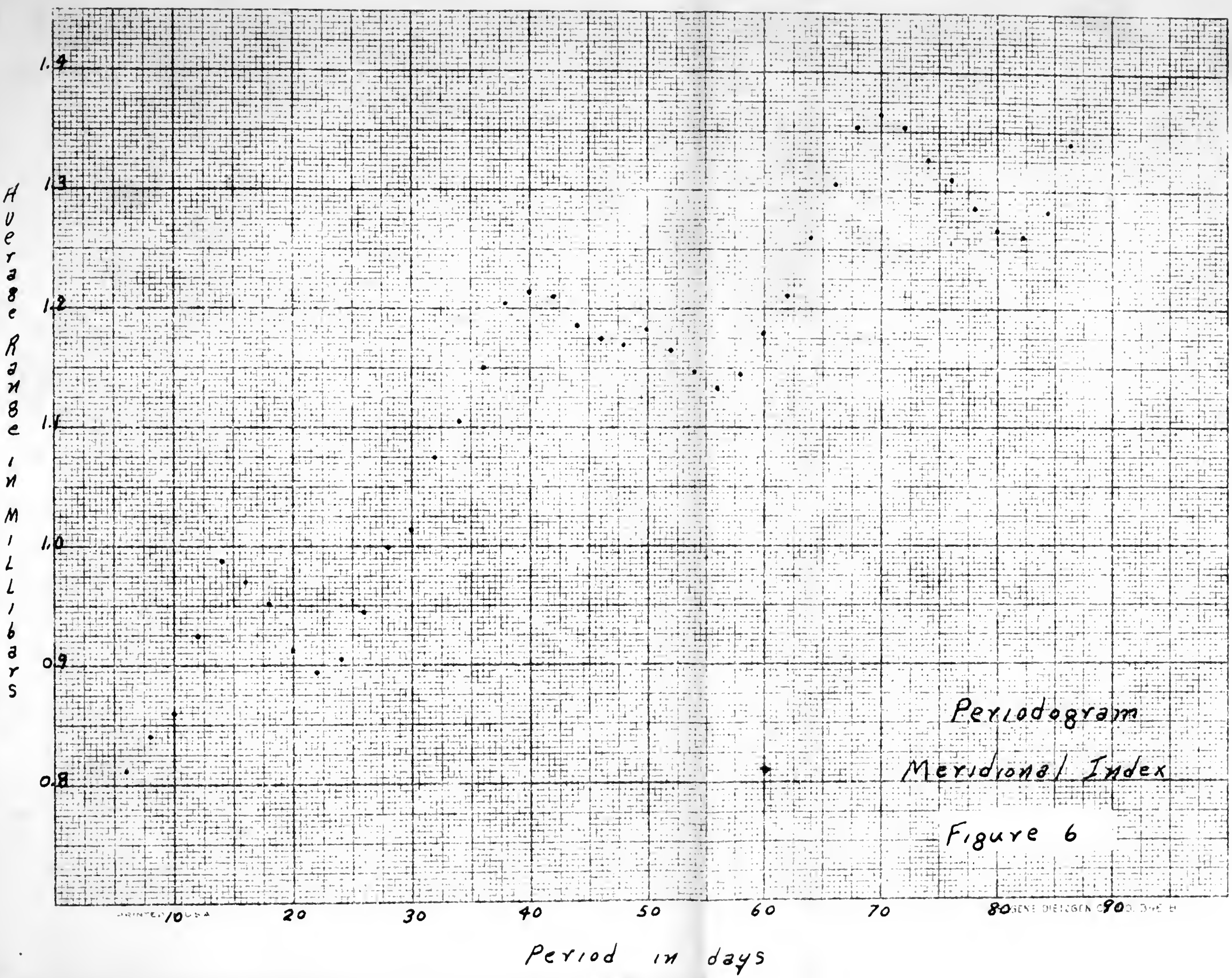
(b) Meridional Index.- This periodogram shows maximum ranges at 14, 40, 50 and 70 days. This suggests a 14-day period, as a regular 14 day wave would exhibit maximum at 14, 42 and 70 days. The explanation for the 50-day maximum is that it is probably the manifestation of a strong even overtone. In view of the previously mentioned accuracy of this method of establishing favored periodicity, and of subsequent results, it is probable that the actual periodicity of this index is somewhat less than 14 days.

(c) Thermal Wind Index.- The periodogram for this index shows maxima at 16, 38, 58 and 84 days. This would seem to indicate a preference for a 12-day basic wave and its odd overtones, as the 12-day wave would exhibit its maximum contribution at 12, 36, 60 and 84 days. In view of the significant correlation between TWI and MI over the complete range of data it is likely that the two indices have the same periodicity. Therefore it is

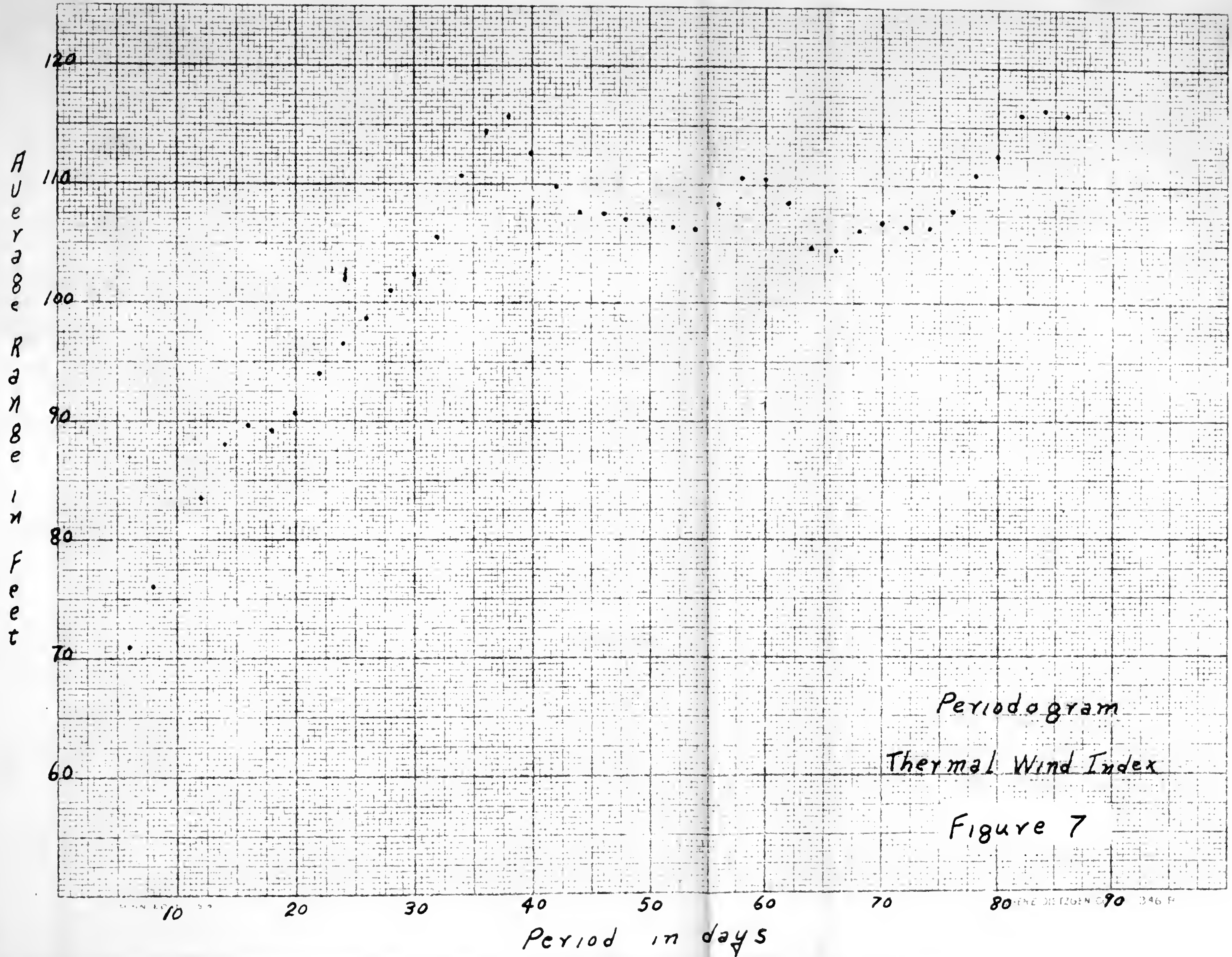


Average Range in Millibars
4.5
4.0
3.5
3.0
2.5
10
20
30
40
50
Period in


Average Range in Millibars
4.5
4.0
3.5
3.0
2.5
10
20
30
40
50
60
70
80
90
Period in days
Periodogram
Zonal Index
Figure 5



Average Range in Millibars
1.4
1.3
1.2
1.1
1.0
0.9
0.8
Periodogram
Meridional Index
Figure 6
10
20
30
40
50
60
70
80
90
Period in days



Periodogram

Thermal Wind Index

Figure 7

believed that a complete harmonic analysis will reveal that TWI and MI have a common basic period and that the value of that period is between 12-14 days.

Now let us state some conclusions obtained from preceding results:

1. The MI and TWI appear to have a common basic period of 12-14 days which would also be the period of HTI.

2. The ZI appears to have a basic period, of about 12 days, which is slightly smaller than the common basic period for the other two indices.

3. This study verifies the existence of "persistent regularities" in the general circulation. For example, Elliott [1] has qualitatively observed "polar cycles" with periods of 25-40 days, whereas Rossby [8] discusses "turbulence waves" with periods of 28-42 days. These periods are consistent with the periods indicated by the second and third overtones of the basic periods, 12-14 days, indicated above.

Now let us examine some possible explanations of the observed correlations in light of the results of the periodicity analysis.

First, it is likely that the basic periodicity of the zonal index is different from that of the other two indices. This would very effectively explain the weak correlations observed and would also explain the failure of any investigator to obtain significant correlations, either contemporary or lag, between daily values of ZI and those of any other index.

In examining another possible explanation of the relatively weak correlations obtained in this study let us consider time-graphs of two indices with the same basic periodicity. Suppose also that these graphs are decomposed into the basic wave and its harmonic overtones. Now if

we investigate the phase-shifts between all comparable harmonics of same period of these two indices, it is highly unlikely that we would find them all identical. It can be easily seen that if these shifts were all identical, and the amplitude ratios between harmonics of the same period remained the same, that a lag correlation of 1.0 could be obtained. However, if the phase-shift between the harmonics were allowed to vary unequally a weakened correlation would result. Carrying this line of reasoning one step further and supposing that the amplitude ratios of the two indices were different, it can be seen that the correlation would be further weakened.

The results of this study seems to indicate that MI and TWI have the same basic periodicity and possibly the same overtone periods, but varying phase-shifts, hence moderately significant correlations result.

The insignificance of the other correlations are explained as the results of phase-shifts, varying amplitude ratios, and possibly differing basic periodicities and harmonics. It therefore appears that the use of linear regression techniques are not very useful in determining significant relationships between the daily values of rapidly fluctuating variates. Instead these methods should be applied to long-term mean values. It would appear that, insofar as daily values are concerned, harmonic analysis techniques might give better results.

BIBLIOGRAPHY

1. Elliott, R. D. Extended Weather Forecasting by Weather Type Methods. U. S. Weather Bureau, 1944.

2. Fisher, R. A. Statistical Methods for Research Workers. Eighth Edition, 1938, p. 202.

3. Jones, M. W. and Jones, J. G. Tidal Effects in the Ionospheric Layer. J. Meteor. 7, 1950.

4. Headquarters, Air Weather Service. Historical Weather Maps. October 1945 - March 1946.

5. Namias, J. and Clapp, P. F. Confluence Theory of the High Tropospheric Jet Stream. J. Meteor. 6, 1949.

6. Namias, J. Extended Forecasting by Mean Circulation Methods. February 1947.

7. Riehl, H., Yeh, T. C., and LaSeur, N. E. A Study of Variations of the General Circulation. Office of Naval Research, December 1949.

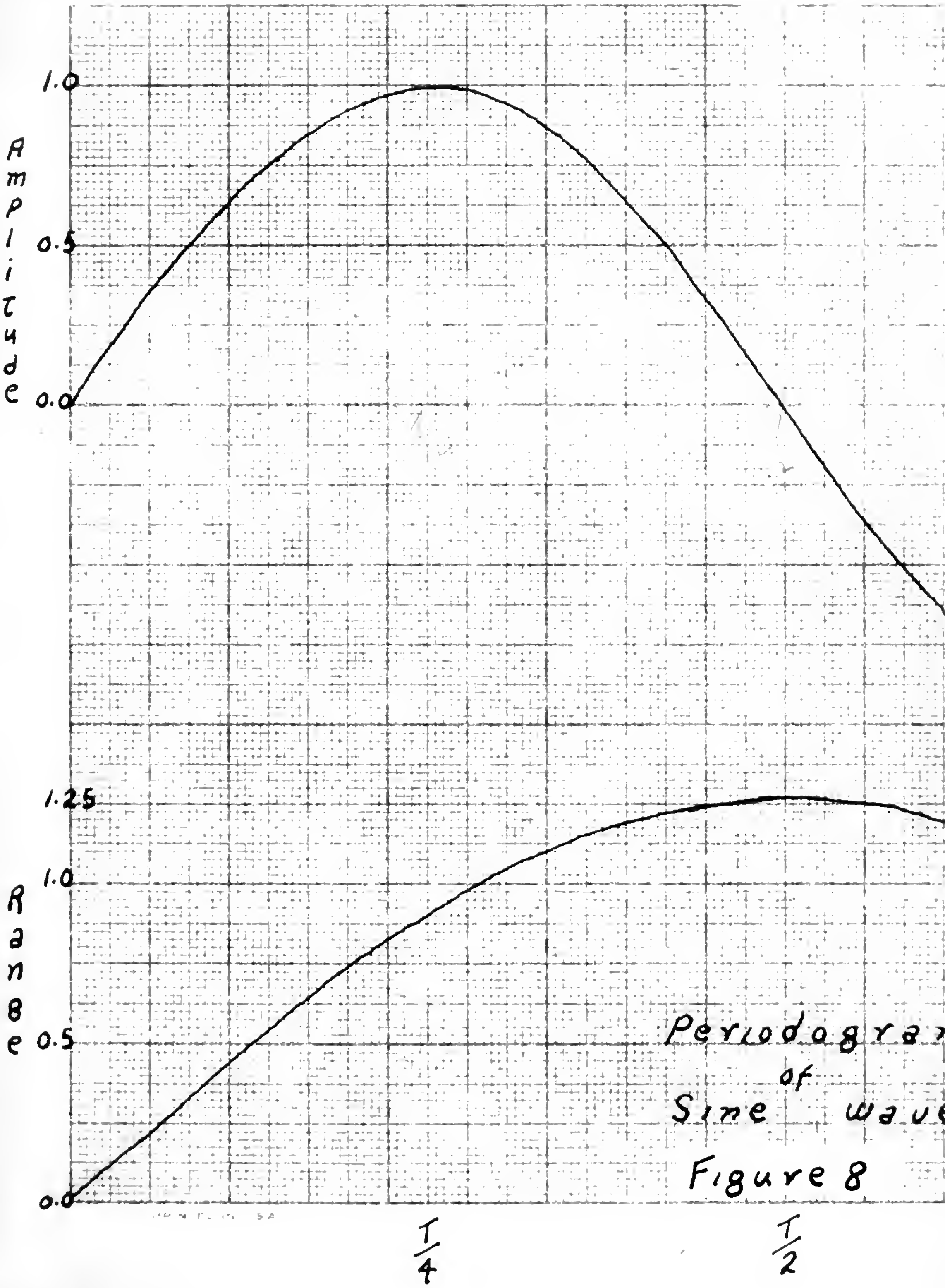
8. Rossby, C. G. Nature of the General Circulation of the Lower Atmosphere. Atmospheres of the Earth and Planets, by Kuiper, 1947.

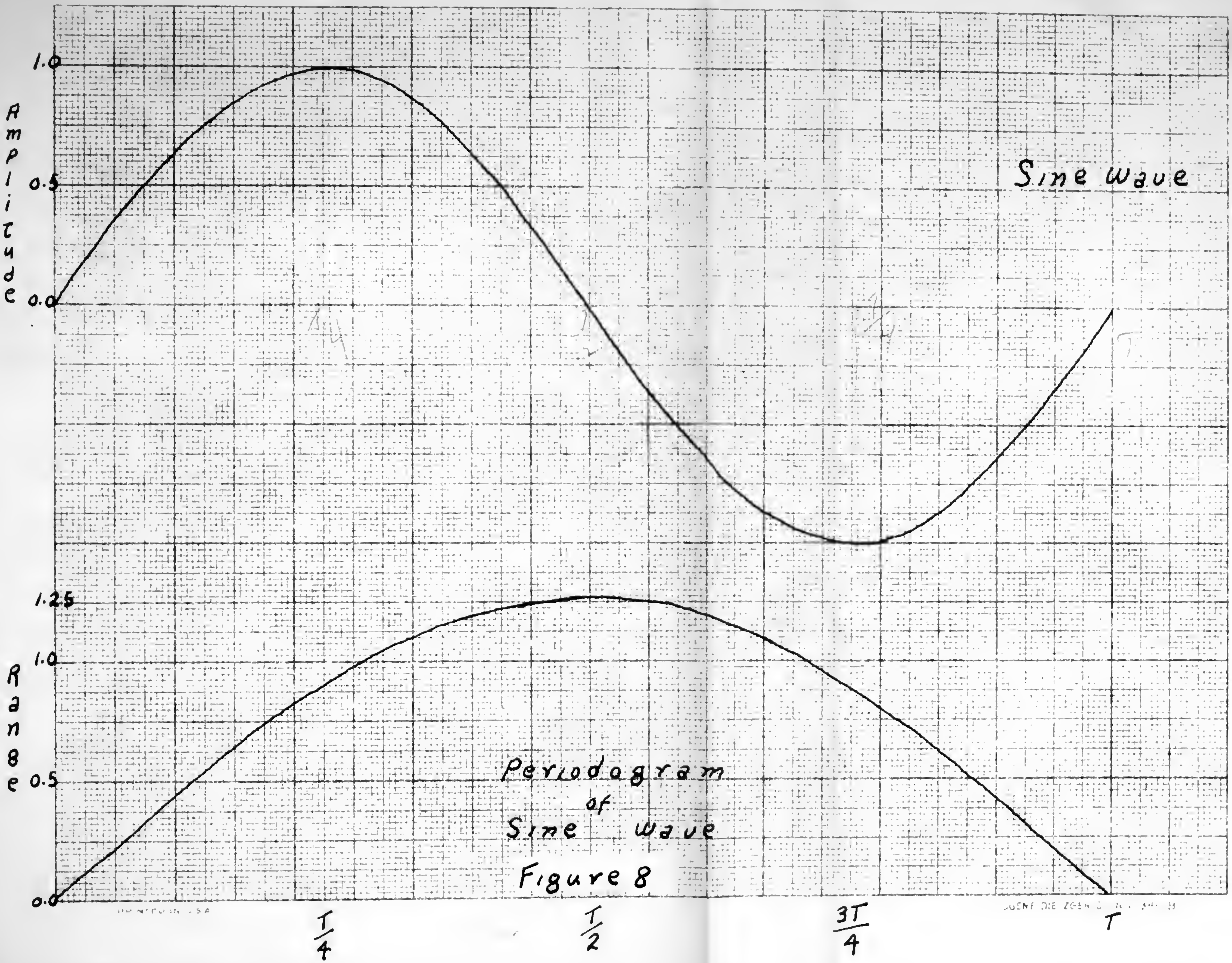
9. Willett, H. C. Extended Forecasting Project 3-12, 1946.

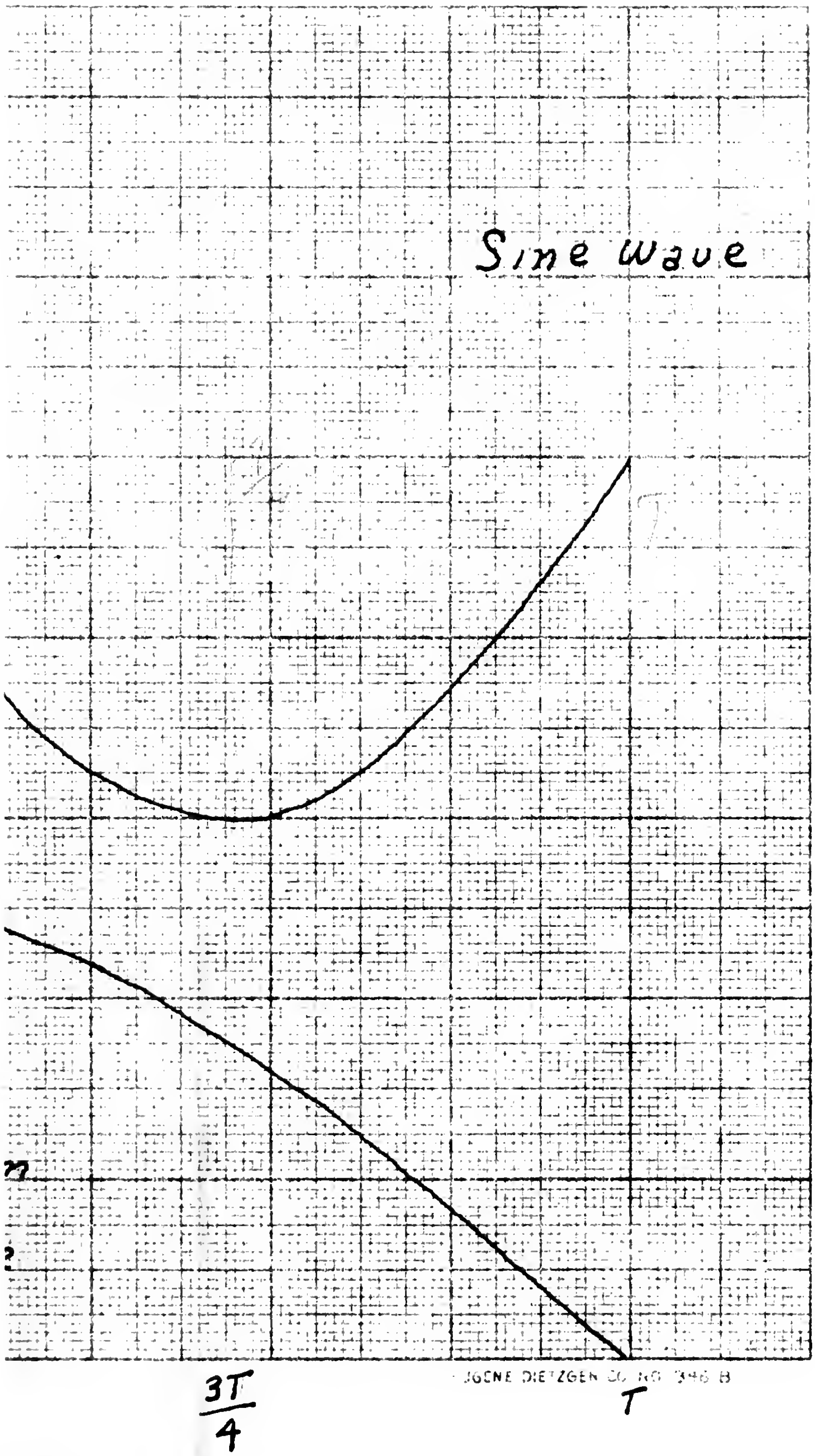
10. Willett, H. C. Patterns of World Wide Weather Changes. American Geophysical Union, Volume 29, 804-805, 1948.

11. Willett, H. C. Significant Correlations between Mendional Heat Transport and the Zonal Character of the General Circulation. J. Meteor. 6, 1949.

[illegible]


1.0
0.5
0.0
Amplitude
1.25
1.0
0.5
0.0
Range
Periodogram
of
Sine wave
T/4
T/2
Figure 8


1.0
0.5
0.0
Amplitude
Sine wave
1.25
1.0
0.5
0.0
Range
Periodogram of Sine wave
T/4
T/2
3T/4
T
Figure 8



Sine wave
3T/4
T
EUGENE DIETZGEN CO. NO. 346 B

APPENDIX II.

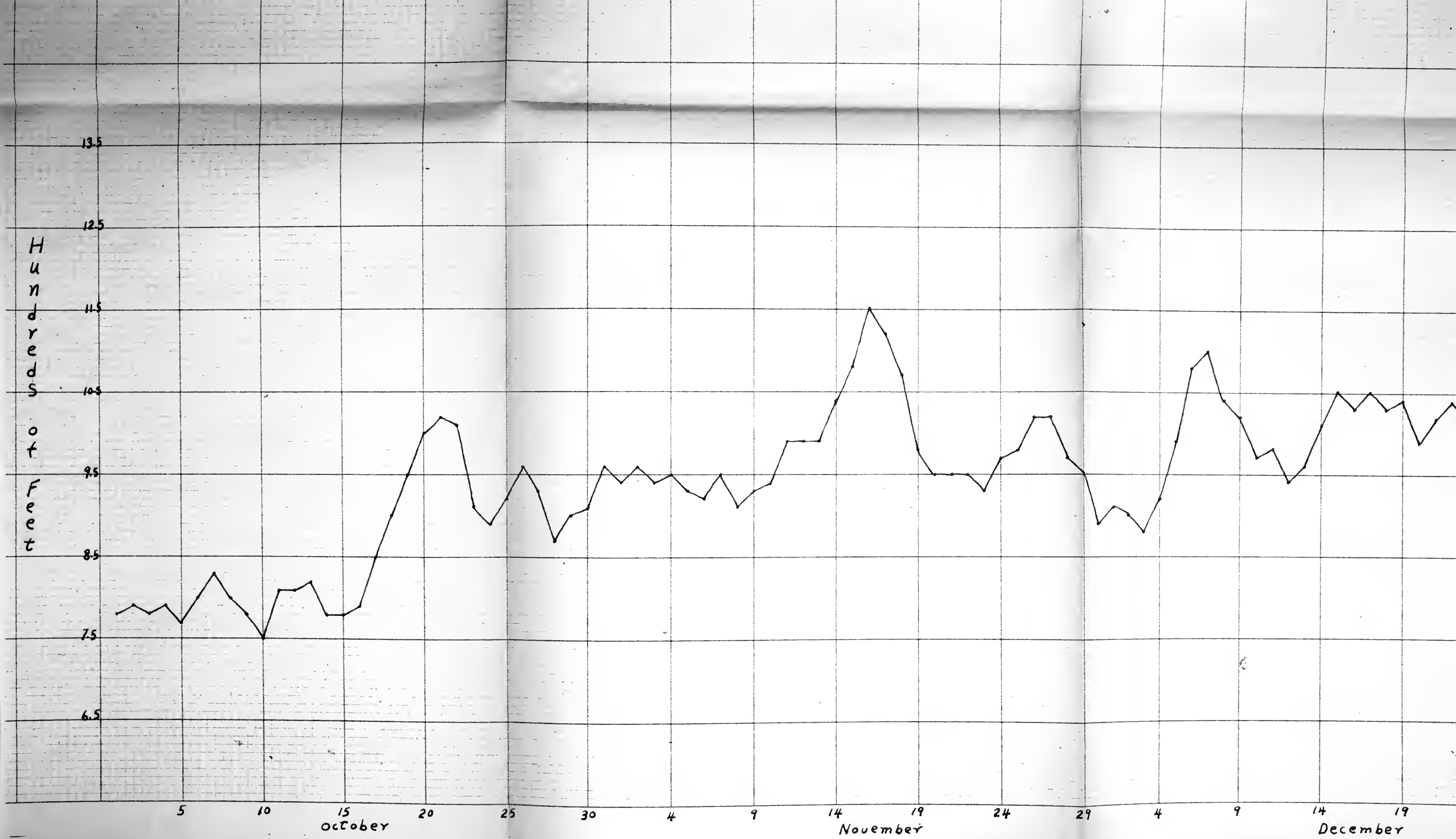
The Periodogram

The periodogram [3] is a rather novel method of determining the most favored periodicity in data whose variation is the sum of many regular waves. It employs the fact that the maximum average range of a regular wave is found between points exactly one-half period apart.

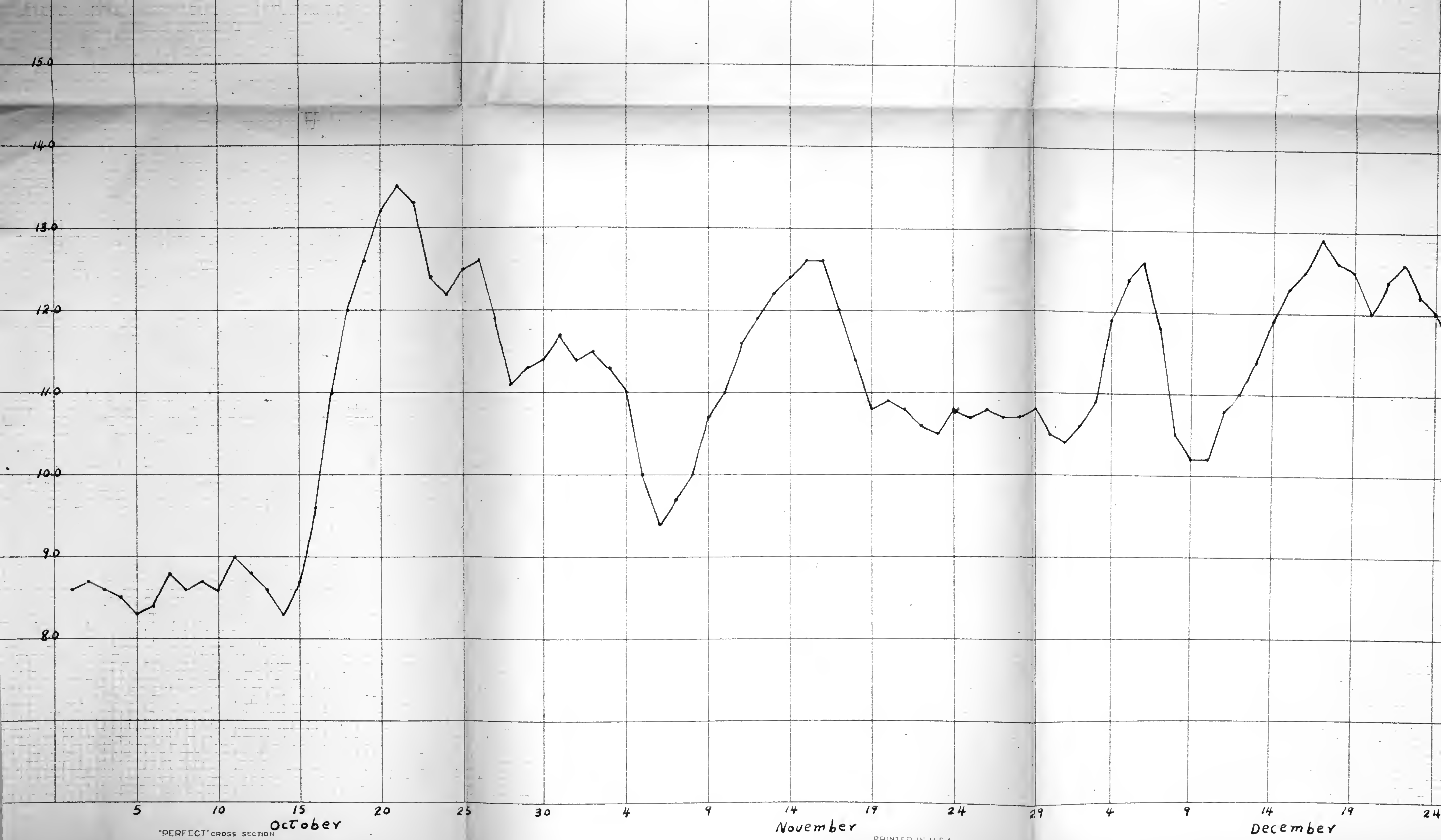
Assume that we have a variate whose fluctuations follow a curve which is the resultant of several superimposed regular waves. If the periodicities of the superimposed components are sufficiently far apart we would, with sufficient data, be able to determine them by computing the average range between observed values of the variate at regular intervals apart.

For instance, if 30 daily values of a variate are taken it is readily seen that in this data there would be 28 values of two-day range; i.e., the absolute difference between the first and third days values, between the second and fourth, etc. If these twenty-eight values were averaged, the average range for the two-day period is obtained. Now if we compute this average range for intervals of 3 days, 4 days, etc., and plot them against a time scale equal to two times the interval, we will have a periodogram. Thus if the periods of the superimposed waves are sufficiently far apart the periodogram will exhibit a maximum at the day corresponding to each. It is to be noted that if data involving several periods of the same wave or waves is analyzed the contribution of this wave or these waves to the exhibited range of the periodogram will be a maximum at all odd multiples of its half period and will fall to zero on all even multiples of its half period.

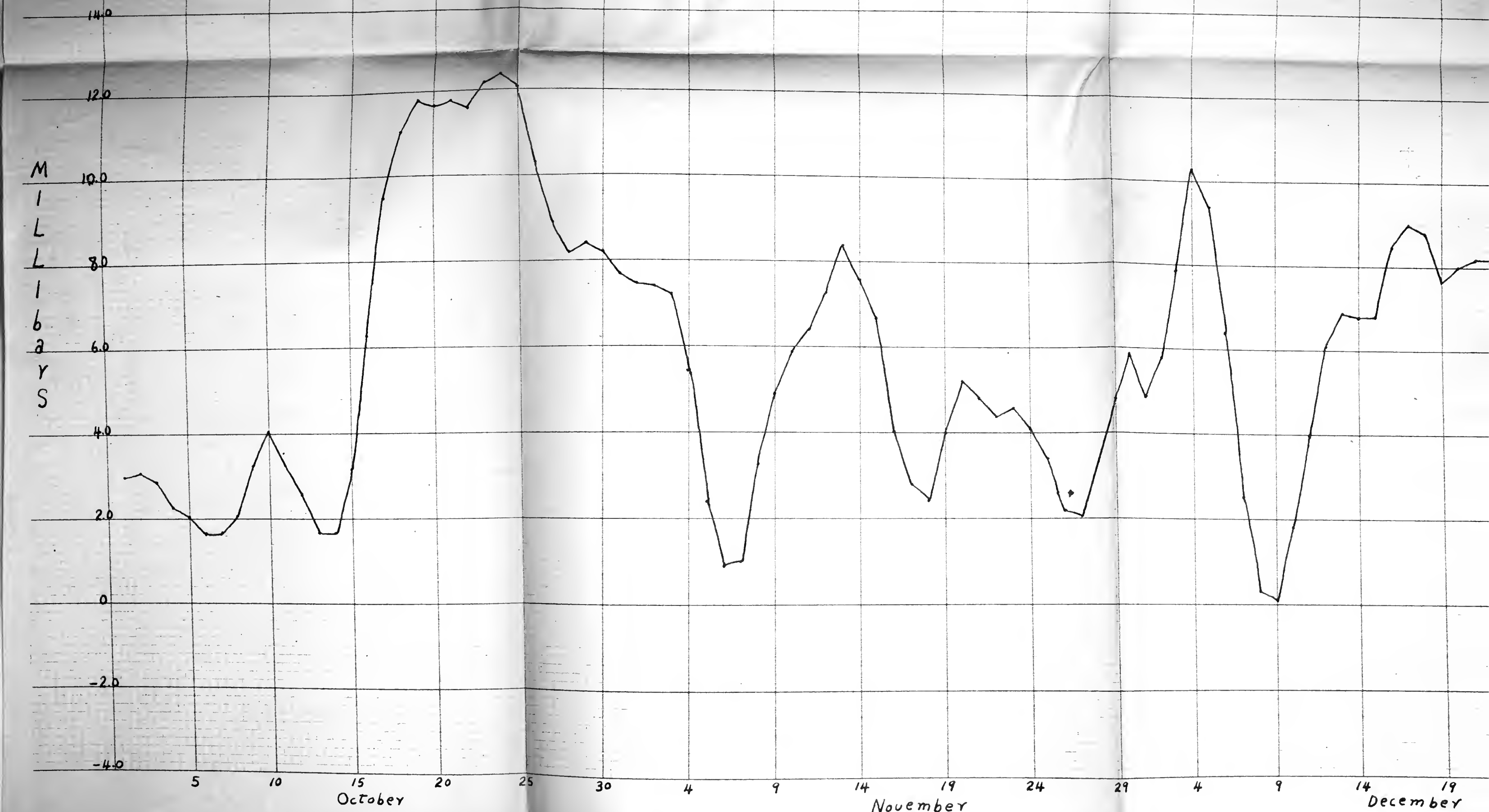
See Figure 8 for an example of a regular wave and its periodogram. Since the time-interval of the maximum range indicates a period exactly double this time-interval, the accuracy of this method is to the nearest two-day interval.



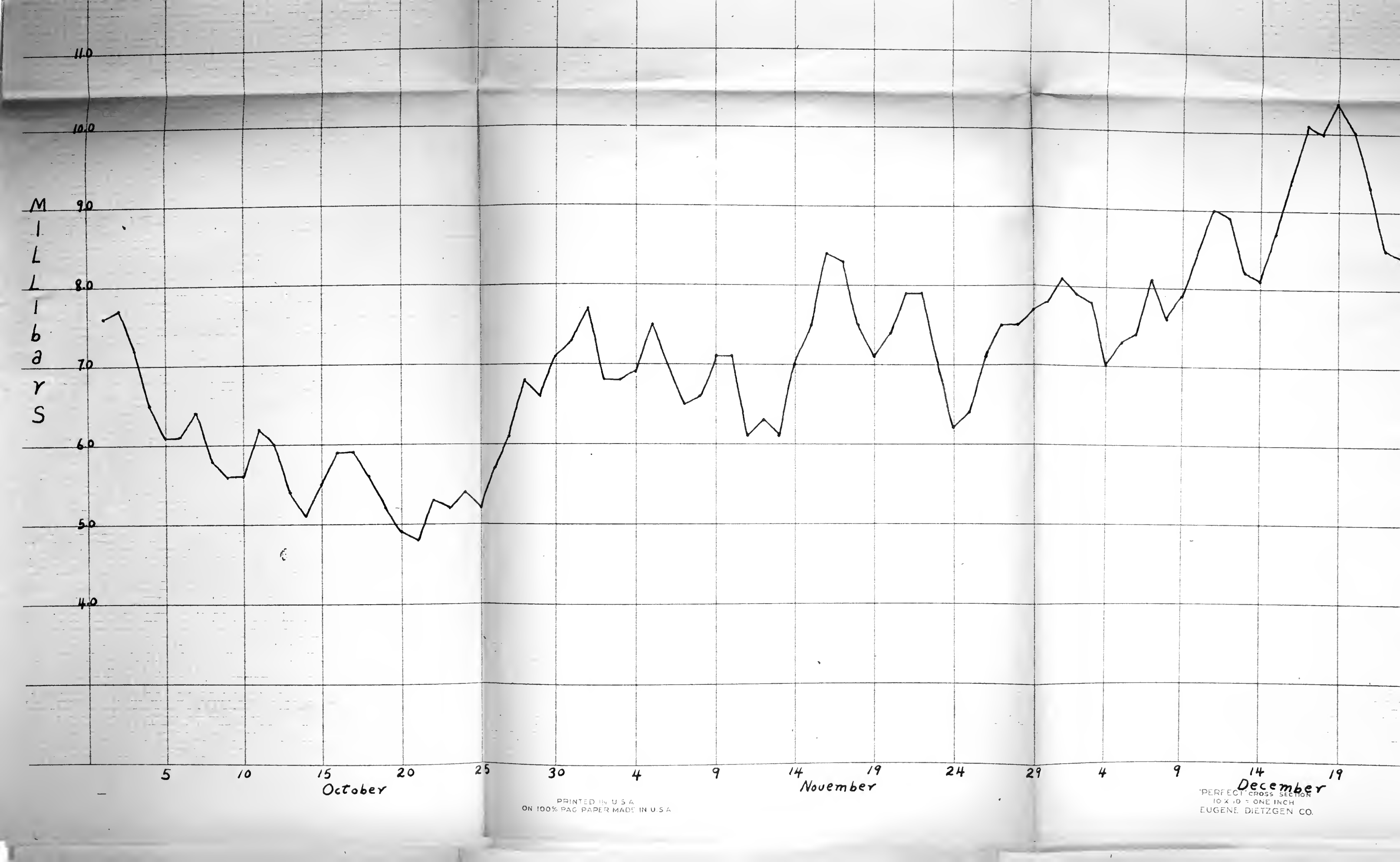
Hundreds of Feet
13.5
12.5
11.5
10.5
9.5
8.5
7.5
6.5
5
10
15
20
25
30
4
9
14
19
24
29
4
9
14
19
October
November
December



"PERFECT" CROSS SECTION
10 X 10 = ONE INCH
EUGENE DIETZGEN CO.
PRINTED IN U.S.A.
ON 100% RAG PAPER MADE IN U.S.A.

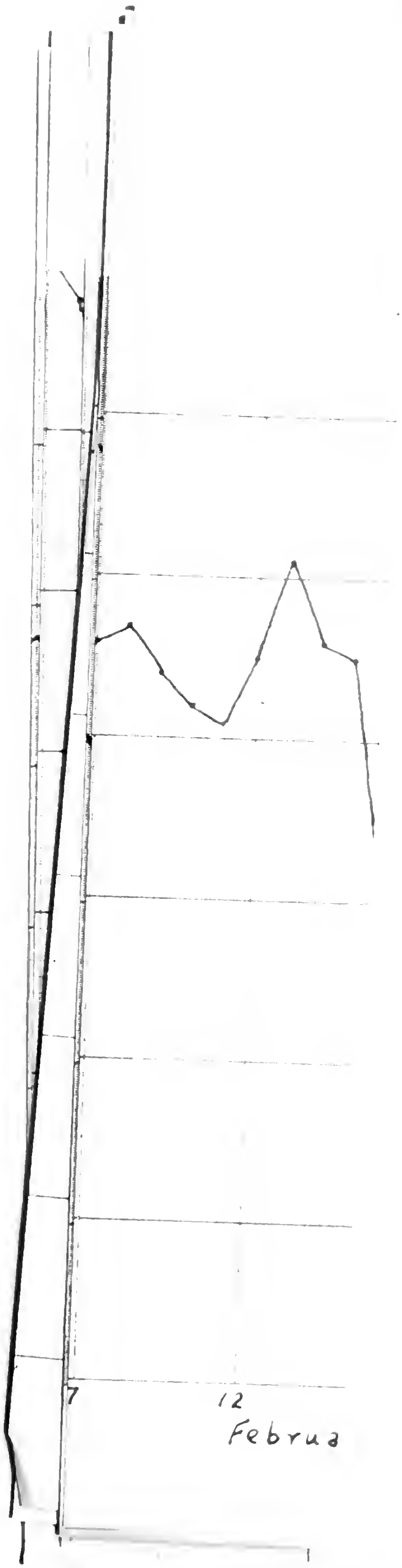


14.0
12.0
10.0
8.0
6.0
4.0
2.0
0
-2.0
-4.0
MILLIBARS
5
10
15
20
25
October
30
4
9
14
19
24
29
November
4
9
14
19
December



M
I
L
L
I
b
a
r
S
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
5
10
15
20
25
October
30
4
9
14
November
19
24
29
4
9
14
December
19
PRINTED IN U.S.A.
ON 100% RAG PAPER MADE IN U.S.A.
"PERFECT" CROSS SECTION
10 X 10 = ONE INCH
EUGENE DIETZGEN CO.

The
B8c
P
c


7
12
Februa


7
12
Februa